ANALYSIS OF ARRHENIUS ACTIVATION ENERGY IN ELECTRICALLY CONDUCTING CASSON FLUID FLOW INDUCED DUE TO PERMEABLE ELONGATED SHEET WITH CHEMICAL REACTION AND VISCOS DISSIPATION

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ABSTRACT

The upfront intention of this study is to explore the advances in electrically conducting Casson fluid induced due to a porous elongated surface taking Arrhenius activation energy, viscous dissipation and joule heating into account. Uniform magnetic and electric fields are imposed on the given flow. Variables of similarity are induced to transmute partial differential equations into dimensionless equations and resolved numerically by elegant method bvp4c. To scrutinize the behavior of critical parameters on flow configurations graphs and table are portrayed. From graphical moments, it is analyzed that velocity of the liquid diminish for advanced values of non-Newtonian rheology parameter, magnetic parameter, porous parameter and inertial parameter. This study also reported that activation energy parameter enhances concentration profiles, whereas fitted rate constant shows opposite behavior. Impact of skin friction, Sherwood and Nusselt numbers on the flow configurations for diverse critical parameters are exposed realistically via graphs.

Key words: Non Newtonian Rheology parameter, porous parameter, magnetic parameter, Casson constitutive equations, Arrhenius equation, activation energy parameter, reaction rate parameter, electrical field parameter.

1. INTRODUCTION

Chemical reaction plays dynamic role in many industrial and technological applications, so there is a great demand to minimize number of reagents and to maximize desired output. Solar energy became the alternative energy source all over the world. But solar technologies have many obstacles like low lifetime of solar cell, small execution balance of structures. Deep insight of activation energy on these sources is helpful to overcome impediments and enlightening the energy stability. The Arrhenius activation energy combined with chemical reaction in MHD flows has countless applications such as chemical engineering, geology, oil refineries, food industries, lubricants so forth. This energy of activation is the quantity of energy that is prerequisite to break down the chemical bonds to start up the chemical reaction. Arrhenius equation best explains transport phenomena under chemical reaction and temperature. Zeeshan et al. (2018) explains the influence of activation energy in Couette Poiseuille of nano fluid. Irfan et al. (2019) studied combined effect of both these energies in 3D Carreau nano fluid by inducing Buongiorno’s theory. Activation energy in peristalsis of Jeffrey fluid is explored by Hayat et al. (2019). Arifuzzaman et al. (2018, 2019) implemented finite difference method to explore influence of chemical reaction on 4th and 2nd grade fluid transport property and MHD flow past porous plates. Hosia (2017) promoted radiation electrical MHD activation energy using Carreau- Nano fluid with parameters control method. Reza-E-Rabbi et al. (2020) enlightened chemical retort on Casson fluid past an elongated sheet using finite difference method and nonlinear chemical reaction effects on multiphase Casson fluid.

The viscosity of the fluid like paints, greases, lubricant oils coal tar, jellies, and paste is not fixed and it depends upon the factors like shear in fluid, pressure and temperature. These fluids are known as non-Newtonian fluids in nature. Casson fluid is non-Newtonian and first proposed by enthusiastic Mathematician Casson (1959) while doing his experiments on letter press toners. The special properties of Casson fluid passed significant applications in science as well as in polymer processing and in biomechanics. Humanoid blood can be measured as Casson fluid because it contains protein, fibrinogen, etc. Inducing Laplace transform technique Hari et al. (2018) obtained exact and numerical solution for unsteady convective Casson fluid considering magnetic field into account. Fazal Mabood et al. (2019) focused porous medium in Casson fluid flow under thermal radiation.

Sulochana et al. (2019) elaborately explained half current result in Casson fluid flow Vijaya et al. (2018, 2019, 2020) comprehensively studied Casson fluid and Casson Nanofluid on stretching surface by considering the impact of radiation, thermophorosis, chemical reaction and Brownian motion. Gayetri et al. (2020) considered Carreau fluid through a stretching sheet with variable thickness and observes that Weissenberg number boosts the velocity. Naga Santhoshi et al. (2020) deliberately discussed three dimensional Casson and Carreau

MHD flow intensifies the curiosity of many researchers now a days for the reason that it plays big role in numerous applications in all fields of science and technology like plasma physics, aerodynamics, astrophysics and super conduction coils. The physical phenomena of viscous flows through porous media can be observed in energy extraction from thermal regions, solid filtration, sedimentary petrology, and also blood stream in the tissue region. MHD flow of Carreau Nano fluid explored using CNT over a nonlinear elongated sheet is studied by Nagalakshmi et al. (2020). Dhana Lakshmi et al. (2019) studied porous effects on MHD convective flows. Sandya et al. (2019) explore MHD flow of inclined porous plate under the influence of radiation and chemical reaction. Kumar et al. (2019) considered slip flow regime with chemical effects in unsteady MHD flow. Vedhavathi et al. (2018) studied effects on MHD convective flows. Vedhavathi et al. (2018) studied porous effects on MHD convective flows. Vedhavathi et al. (2018) studied porous effects on MHD convective flows. Vedhavathi et al. (2018) studied porous effects on MHD convective flows. Vedhavathi et al. (2018) studied porous effects on MHD convective flows. Vedhavathi et al. (2018) studied porous effects on MHD convective flows. Vedhavathi et al. (2018) studied porous effects on MHD convective flows. Vedhavathi et al. (2018) studied porous effects on MHD convective flows. Vedhavathi et al. (2018) studied porous effects on MHD convective flows. Vedhavathi et al. (2018) studied porous effects on MHD convective flows.

In the above constitutive equations $\tau_{ij}$ is the $(i, j)^{th}$ stress tensor component, $\mu_B$ is the plastic dynamic viscosity of the fluid which is non-Newtonian. When $P_y > P_s$ fluid acts as a solid, and when $P_y < P_s$ fluid demonstrates flow characteristics, where $P_y$ is yield stress, $P_s$ is shear stress. $\pi_c$ is the critical value of $\pi = e_y e_j (e_j (i, j)^{th}$ component of deformation rate) which depends upon non-Newtonian model. The governing equations of the flow are as rearranged here under with terms mentioned in nomenclature.

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$$

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = v \left(1 + \frac{1}{\beta} \right) \frac{\partial^2 u}{\partial y^2} + \sigma \left(E_x B_0 - B_0^2 \right) - \frac{C_p}{\sqrt{\kappa P}} \frac{\partial^2 u}{\partial y^2}$$

$$-\frac{\nu}{\kappa} \frac{\partial u}{\partial y} + g \beta T (T^* - T_\infty) + g \beta c (C^* - C_\infty)$$

$$\frac{\partial T^*}{\partial x} + \frac{\partial T^*}{\partial y} = k \frac{\partial^2 T^*}{\partial y^2} + \frac{16 \alpha T_\infty^3}{3 \kappa \rho C_p} + \frac{\mu}{2} \left(1 + \frac{1}{\beta} \right) \frac{\partial^2 u}{\partial y^2},$$

$$+ \frac{\sigma}{\rho C_p} (u B_0 - E_0)^2$$

$$\frac{\partial C^*}{\partial x} + \frac{\partial C^*}{\partial y} = \beta \frac{\partial^2 c^*}{\partial y^2} + \frac{\partial k p}{\partial y} \frac{\partial^2 p}{\partial y^2} - k^2 (C^* - C_\infty) \left(\frac{C^*}{C_\infty}\right)^n e^{-\frac{e (r^* - r_\infty)}{T_\infty}}$$

In the above governing equations (3&4), $\beta(= \mu_B \sqrt{2 \pi c / P_s})$ is the Casson parameter, and in equation (5) the term $k^2 \left(\frac{C^*}{C_\infty}\right)^n e^{-\frac{e (r^* - r_\infty)}{T_\infty}}$ represents the modified Arrhenius equation in which the reaction rate is given by $k^2$, the activation energy is given by $E_a$, the Boltzmann constant is given by $\xi = 8.61 \times 10^{-5}$ $eV / K$, and the fitted rate constant is given by ‘n’ which lies between -1 and 1. Soret effect is also studied through the term $\frac{\partial k p}{\partial y} \frac{\partial^2 p}{\partial y^2}$ in the equation (5). The associated boundary conditions with stretching surface velocity is $u_w (x) = b x$, $T_w > T_\infty$, $C_w > C_\infty$ where proportionality constant ‘b’ are as follows.

$$\begin{align*}
\frac{\partial u}{\partial y} & \to 0, \\
\frac{\partial \nu}{\partial y} & \to 0 \\
T^* & \to T_\infty \\
C^* & \to C_\infty
\end{align*}$$

3. SUITABLE CONVERSION

Following likeness variables are introduced to transform PDE (2-5) into ODE

$$\bar{u} = b x f^*(\eta), \quad \bar{v} = -\sqrt{\nu} f(\eta), \quad \eta = \sqrt{\nu} y$$

Fig. 1 Geometrical outline of the flow

A two dimensional incompressible steady Darcy-Forchheimer mixed convective electrically conducting Casson fluid flow induced by a vertically elongated sheet inserted in a fluid saturated porous medium is presented in this study. In this flow geometry, origin is taken as the fixed point and the pane is elongated along x-axis as well as y-axis. Two identical and reverse forces are applied to stretch the sheet. Maxwell’s equation $\nabla \cdot B = 0$, $\nabla \times E = 0$, and Ohm’s law $\mathcal{J} = \sigma (\mathcal{E} + \vec{q} \times \vec{B})$ are measured in the precise flow, with $\vec{B}$, $\mathcal{E}$, $\mathcal{J}$, $\sigma$, $\vec{q}$ are transverse magnetic field, transverse electric field, Joule current, magnetic permeability and fluid velocity respectively. The very slight induced magnetic field generated due to electrical fluid is ignored in this study.

Casson fluid is non-Newtonian in nature and its constitutive equations (Eldebe and Salwa, 1995) are written as follows.

for $\pi > \pi_c$ , $\tau_{ij} = 2 (\mu_B + P_y (2 \pi_c)^{\frac{1}{2}}) e_{ij}$

for $\pi < \pi_c$ , $\tau_{ij} = 2 (\mu_B + P_y (2 \pi_c)^{\frac{1}{2}}) e_{ij}$

(1)
\[
\theta(\eta) = \frac{v' - v_0}{v_0 - v_\infty}, \quad \phi(\eta) = \frac{C^r - C_\infty}{C_0 - C_\infty}, \quad (8)
\]

Substituting the likeness variables defined in equation (7&8) into the governing equations (2) – (5) we obtain

\[
\left(1 + \frac{1}{\beta}\right)f'''' + ff'' - f'^2 + M(E_1 - f') - F_e f'' - D f' + R(\theta + N\phi) = 0 \tag{9}
\]

\[
\left(1 + \frac{4}{3}N\right)\theta'''' + Pr \left[\frac{f'}{1 + \frac{1}{\beta}} Ec \left(E f''ight)^2\right] = 0 \tag{10}
\]

\[
\phi'' + Sc(f\phi' + Sr \theta'') - Sc \Lambda (1 + \delta \theta)^n e\left(-\frac{\beta \theta}{1 + \delta \theta}\right) = 0 \tag{11}
\]

\[
\begin{aligned}
& f(0) = 0 \\
& f'(0) = 1 \quad \text{at } \eta = 0 \quad \left\{ f'(\infty) = 0 \right. \\
& \theta(0) = 1 \quad \phi(\infty) = 0
\end{aligned} \tag{12}
\]

Here \(M = \frac{\alpha \Phi}{\beta \rho b}\) the magnetic parameter, \(R = \frac{Gr}{Re^2}\) the thermal buoyancy number, \(N = \frac{Gr}{Gr_\infty}\) the solutal buoyancy number, \(Gr = \frac{g \beta (T_\infty - T_0) \lambda^2 \epsilon}{\nu^2}\) the Grashof number due to temperature, \(E_1 = \frac{g \beta (C_\infty - C_\infty) \lambda^2}{\nu^2}\) the Soret number, \(\Lambda = \frac{\nu}{\eta b x}\) the reaction rate parameter, \(E = \frac{k_0}{\epsilon (C_\infty - C_\infty)}\) the activation energy parameter, \(\delta = \frac{T_\infty - T_0}{T_0}\) the temperature difference parameter.

\section*{4. PHYSICAL QUANTITIES}

The significant engineering physical quantities in this problem are \(C_f\) (skin friction coefficient), \(Nux\) (local Nusselt number), \(S_h\) (local Sherwood number) respectively are defined below with \(Re_x = \frac{u_x X}{v}\) as local Reynolds number

\[
C_f = \frac{2 \tau_w}{\rho u_0^2}, \quad Nux = \frac{x q_0}{k(T_\infty - T_0)} \quad \text{and} \quad S_h = \frac{x q_m}{\beta_m(C_0 - C_\infty)}, \tag{13}
\]

\[
\tau_w, q_0, q_m \text{ are wall shear stress, heat transfer, mass transfer respectively and are given by}
\]

\[
\tau_w = \mu \left(1 + \frac{1}{\beta}\right) \left(\frac{\partial \theta}{\partial \eta}\right)_{\eta=0} \tag{14}
\]

\[
q_0 = -R \left(\frac{\partial \theta}{\partial \eta}\right)_{\eta=0} \tag{15}
\]

\[
q_m = -D \left(\frac{\partial C}{\partial \eta}\right)_{\eta=0} \tag{16}
\]

Applying the variable from equation (7), using equations (14-16) in (13) we obtain,

\[
\frac{1}{2} C_f \sqrt{Re_x} = \left(1 + \frac{1}{\beta}\right)f''''(0) \tag{17}
\]

\[
Nu_x Re_x^{1/2} = -\left(1 + \frac{4}{3} N\right)\theta'(0) \tag{18}
\]

\section*{5. CONFIRMATION OF NUMERICAL UPSHOTS}

The moderate bvp4c with technique of shooting is applied to explain the highly nonlinear joined equations [8, 9 & 10] along with the associated boundary conditions [11]. Table.1 shows the assessment of present outcomes with that of previous outcomes, where we can observe exact agreement.

\begin{table}[h]
\centering
\begin{tabular}{|c|c|c|c|c|c|}
\hline
\hline
0 & 1.00000 & 1.000008 & 1.000008 & 1.000007 \\
1 & 1.4142 & 1.4142 & 1.4142 & 1.4142 \\
5 & 2.4494 & 2.4494 & 2.4494 & 2.4494 \\
10 & 3.31662 & 3.31662 & 3.31662 & 3.31662 \\
50 & 7.14142 & 7.14142 & 7.14142 & 7.14142 \\
100 & 10.04987 & 10.04987 & 10.04987 & 10.04987 \\
500 & 22.38380 & 22.38380 & 22.38380 & 22.38381 \\
1000 & 31.63858 & 31.63858 & 31.63858 & 31.63858 \\
\hline
\end{tabular}
\caption{Assessment values of \(-f''(0)\) for various values of M when \(\beta = F = D = R = N = E_1 = E = 0\)}
\end{table}

\section*{6. GRAPHICAL ILLUSTRATIONS}

\subsection*{6.1. Performance of \(\beta, M\) and \(E_1\) on \(f'(\eta)\), \(\theta(\eta)\), \(\phi(\eta)\)}

The effect of non –Newtonian rheology parameter \(\beta\), magnetic field parameter \(M\) and electrical field parameter \(E_1\) can be observed from Fig. 2(a-f). \(\beta\) accelerates fluid’s plastic dynamic viscosity and resists the fluid flow, as a result gradual fall in velocity can be observed in Fig. 2(a). Advanced numerical values of \(\beta\) increases wideness of the boundary layers in the fluid as a consequence there is gradual increase in...
in temperature and concentration which is shown in Fig. 2(b) & Fig. 2(c). From Figures 2 (d-f) it is witnessed that as M reaches higher values there is a quick reduction in velocity due to Lorentz force which compete against the motion of the fluid, at the same time there is gradual increment in both thermal and solutal boundary layers. Larger value of $E_1$ serves as accelerating force to overcome frictional resistance in the fluid and increases velocity, where there is reduction in temperature and species concentration due to increase in mass flux.

6.2. Performance of $R, N, D, Fr$ on $f'(\eta), \theta(\eta), \phi(\eta)$

The influence of thermal buoyancy parameter ($R$), solutal buoyancy parameter ($N$), permeability parameter ($D$), and inertial parameter ($Fr$) on $f'(\eta), \theta(\eta), \phi(\eta)$ can be observed from Figures 3(a-l). Higher values of $R$ accelerates velocity of the fluid where as temperature and concentration shows reverse effect for advanced values of $R$. Exactly similar effect on these gradients can be observed for $N$. The velocity distribution shows a decreasing tendency for higher value of $D$ and it shows reverse effect on both. It is observed that $Fr$ decreases velocity profile and increases temperature and concentration profiles.
Fig. 3(d) Dominance of N on $f'(\eta)$

Fig. 3(e) Dominance of N on $\theta(\eta)$

Fig 3(f). Dominance of N on $\phi(\eta)$

Fig. 3(g) Dominance of D on $f'(\eta)$

Fig. 3(h) Dominance of D on $\theta(\eta)$

Fig. 3(i) Dominance of D on $\phi(\eta)$

Fig. 3(j) Dominance of $F_r$ on $f'(\eta)$

Fig. 3(k) Dominance of $F_r$ on $\theta(\eta)$
6.3 Performance of Pr, Ec, Nr on $\theta(\eta)$

The performance of Prandtl number (Pr), Eckert number (Ec), thermal radiation parameter (Nr) can be observed in Figures 4(a-c). Comparing to conduction convection plays dominant role in boundary layer in transforming energy, this physical phenomena can be attributed from Fig. 4(a). For higher values of Pr more reduction in temperature is seen. Higher Ec values enhance temperature due to release of stored energy which is illustrated in Fig. 4(b). Larger value of Nr amplifies radiation and it leads to hike in temperature as shown in Fig. 4(c).

6.4 Performance of Sc, Sr, $\Lambda$, $\Delta$, $E$ and $n$ on $\phi(\eta)$

The performance for Schmidt number (Sc), Soret number (Sr), reaction rate parameter ($\Lambda$), temperature difference parameter ($\Delta$), activation energy parameter (E) and fitted rate constant (n) on species concentration are portrayed in Figures 5(a-f). Wideness of the solutal boundary layer becomes thinner when Sc is large as a result concentration reduces. Sr appreciable enhancement on concentration...
distribution. \( \delta \) Diminish concentration. As we improve the values of \( \Lambda \) there is higher destructive amount of chemical reaction in the fluid and reduces concentration. The impact of \( E \) and \( n \) depicted in these graphs. It can be observed that enhancing value of \( E \) amplifies concentration, but fitted rate constant declines it.

6.5 Influence of critical parameters on physical quantities

Dominance of electrical field parameter (\( E_1 \)) along with magnetic field parameter (\( M \)) is portrayed in Fig. 6(a). When \( M \) assumes higher values skin friction coefficient decreases and \( E_1 \) shows opposite behavior, this is because of the Lorentz force produced due to the magnetic field parameter. Both thermal and solutal buoyancy parameters (\( R, N \)) are found to enhance skin friction which is depicted in Fig. 6(b). Both porous parameter and inertial parameter (\( D, Fr \)) decreases surface drag coefficient, as higher values of suction parameter reduce the force employed on the boundary layer of the fluid as illustrated in Fig. 6(c). From Fig. 6(d), it is noted that aggregate values of fitted rate constant \( n \) increases the Sherwood number in very marginal difference. Whereas reaction rate parameter (\( \Lambda \)) shows augmenting performance on it. Intensifying values of activation energy parameter (\( E \)) slows down the. It is pragmatic that high Prandtl number (\( Pr \)) increases Nusselt number. It is also seen that higher values of \( Nr \) has small influence on Nusselt number and Ec number decreases its value.
7. CONCLUSIONS

The present communication inspects the impact of Arrhenius activation energy along with chemical reaction of electrically conducting Casson fluid under the influence of transverse magnetic field, thermal diffusion induced due to porous stretching sheet. The impact of critical parameters on the flow configurations is graphically accessible using bvp4c shooting technique. The strategic consequences of the framework are defined below.

- The fluid velocity \( f(\eta) \) enhanced for the progressive values of activation energy parameter \( E \), Thermal buoyancy parameter \( R \), Solutal buoyancy parameter \( N \), where conflicting tendency is noted for Non-Newtonian Rheology parameter \( \beta \), magnetic parameter \( M \), and permeability parameter \( D \).
- Casson fluid temperature heightened for larger values of \( \beta \), \( M \), \( D \), \( Ec \), \( Nr \) where as temperature decreased for higher values of \( E1 \), \( R \), \( N \), \( Pr \).
- Enhancement in Schmidt number \( Sc \), Chemical reaction parameter \( A \), Activation energy parameter \( E \), temperature difference parameter \( \delta \) produced thinner boundary layers with a reduction in concentration.
- When fitted rate constant ‘n’ changes from \( n=-1 \) to \( n=2 \) a gradual reduction in concentration is observed.
- Results of the analysis are found to be in an excellent with that of previous values.

NOMENCLATURE

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
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<tbody>
<tr>
<td>( k^* )</td>
<td>Absorption coefficient</td>
</tr>
<tr>
<td>( g )</td>
<td>Acceleration due to gravity</td>
</tr>
<tr>
<td>( \bar{u} )</td>
<td>Component of velocity along x - axis</td>
</tr>
<tr>
<td>( C_{fr} )</td>
<td>Concentration in the free stream</td>
</tr>
<tr>
<td>( C^* )</td>
<td>Concentration of the fluid</td>
</tr>
<tr>
<td>( \bar{v} )</td>
<td>Component of Velocity along y - axis</td>
</tr>
<tr>
<td>( C_B )</td>
<td>Drag coefficient</td>
</tr>
<tr>
<td>( E_B )</td>
<td>Electric field strength in transverse direction</td>
</tr>
<tr>
<td>( \nu )</td>
<td>Kinematic viscosity</td>
</tr>
<tr>
<td>( B_{fr} )</td>
<td>Magnetic field strength in transverse direction</td>
</tr>
<tr>
<td>( D )</td>
<td>Mass diffusivity</td>
</tr>
<tr>
<td>( T_m )</td>
<td>Mean fluid temperature</td>
</tr>
<tr>
<td>( k_p )</td>
<td>Permeability of the porous medium</td>
</tr>
<tr>
<td>( T_{fr} )</td>
<td>Temperature in the free stream</td>
</tr>
<tr>
<td>( T_w )</td>
<td>Uniform temperature at the wall</td>
</tr>
<tr>
<td>( c_p )</td>
<td>Specific heat at constant pressure</td>
</tr>
<tr>
<td>( T^* )</td>
<td>Temperature of the fluid</td>
</tr>
<tr>
<td>( k )</td>
<td>Thermal conductivity of the medium</td>
</tr>
<tr>
<td>( k_T )</td>
<td>Thermal-diffusion ratio</td>
</tr>
<tr>
<td>( C_w )</td>
<td>Uniform concentration at the wall</td>
</tr>
</tbody>
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Greek symbols

\[ \beta_c \quad \text{Concentration expansion Coefficient} \]
\[ \rho \quad \text{Density of the fluid} \]
\[ \mu \quad \text{Dynamic viscosity of the fluid} \]
\[ \sigma^* \quad \text{Stefan-Boltzmann constant} \]
\[ \beta_T \quad \text{Thermal expansion Coefficient} \]

REFERENCES


